A new environment to simulate the dynamics in the close proximity of rubble-pile asteroids

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Introduction

Motivation and research goal

Small body close proximity environment

- Uneven mass distribution
  - irregular gravity field
  - internal voids and high porosity (rubble pile)
- Weak gravity field, non-gravitational effects are relevant
  - SRP
  - gas ejecta and coma (active asteroids and comets)
- Orbiting dust and particles
- Granular surface (boulders and pebbles)

Credits: JAXA/Hayabusa   Credits: NASA/NEAR
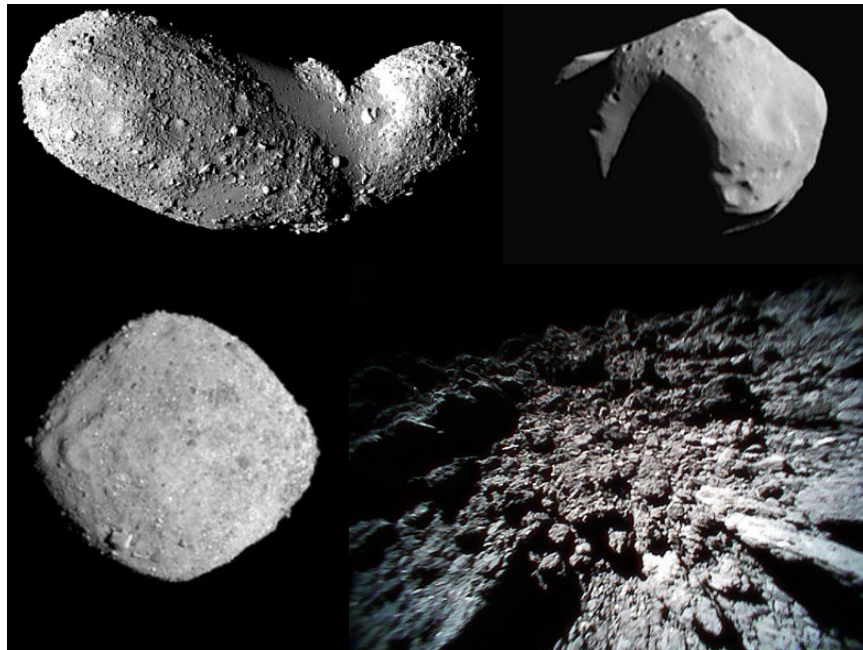
Credits: NASA/OSIRIS-Rex   Credits: JAXA/Hayabusa 2/Minerva-II1-B

Research goal:
Simulate the environment near rubble pile objects, including granular dynamics and non-gravitational effects

Implementation and methods

Software architecture

N-body gravitational problem with contact and collisions

Gravitational dynamics	Contact dynamics

- N-body self-gravity (point mass sources)
  - direct N-to-N integration
  - Barnes-Hut octree (CUDA/GPU parallel)
- central field (shape-based model)
  - Polyhedron (mesh)

- 6 DOF rigid body dynamics
- bodies of arbitrary shape
- collision detection
- contact methods
  - hard-body, constraint-based
  - soft-body, penalty-based
  - constraint-based with compliance and damping

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Implementation and methods

Software architecture

N-body gravitational problem with contact and collisions

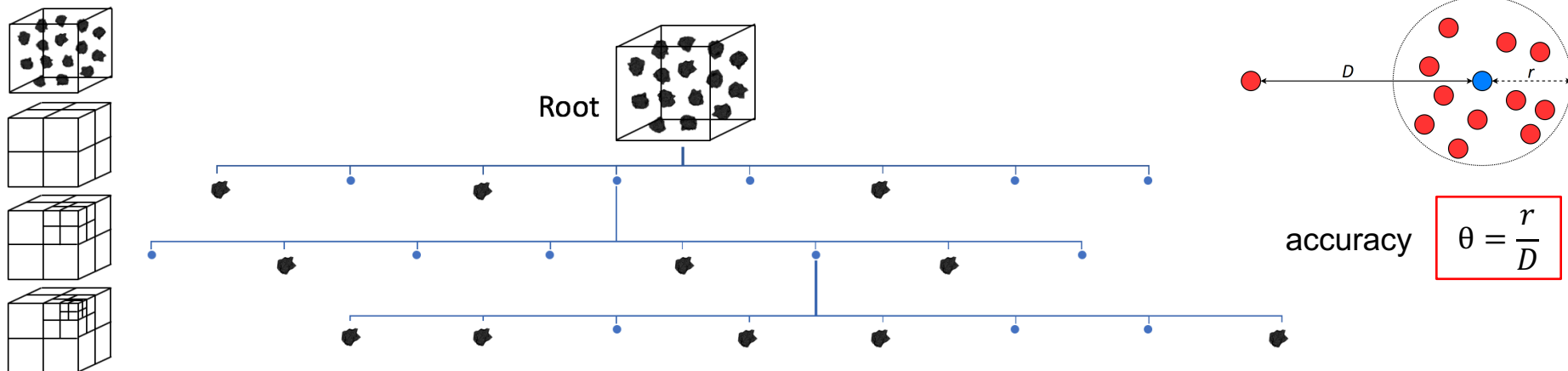| Gravitational dynamics |
|---|

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Contact dynamics

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Implementation and methods

Gravitational dynamics: Barnes-Hut octree (CUDA/GPU parallel)

Root

$$\theta = \frac{r}{D}$$

accuracy

- **Nodes** correspond to **cubes** in the physical space
- Homogenous Spatial Recursive sub-division (until each extremal node has 1 or 0 particles)
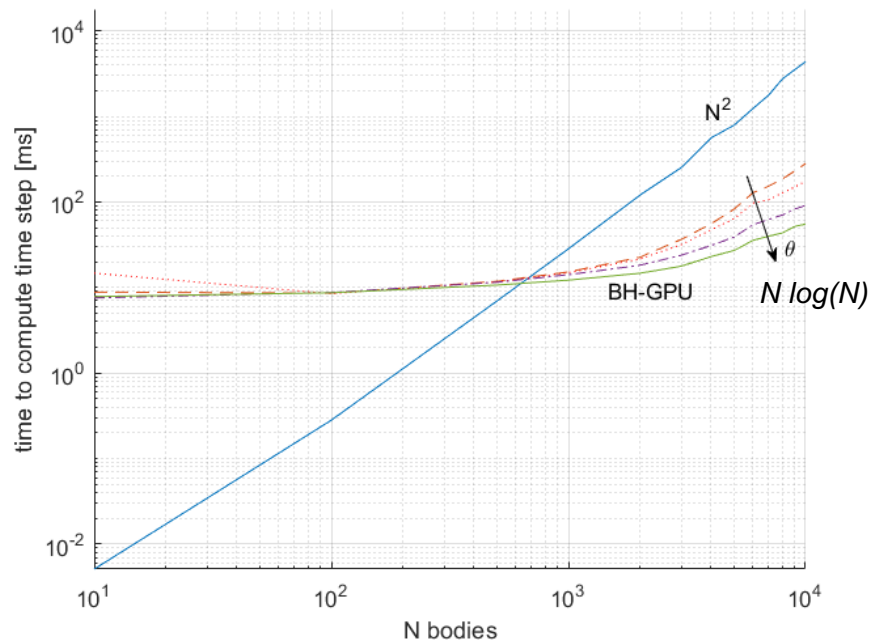- Based on the work by M. Burtscher and K. Pingali

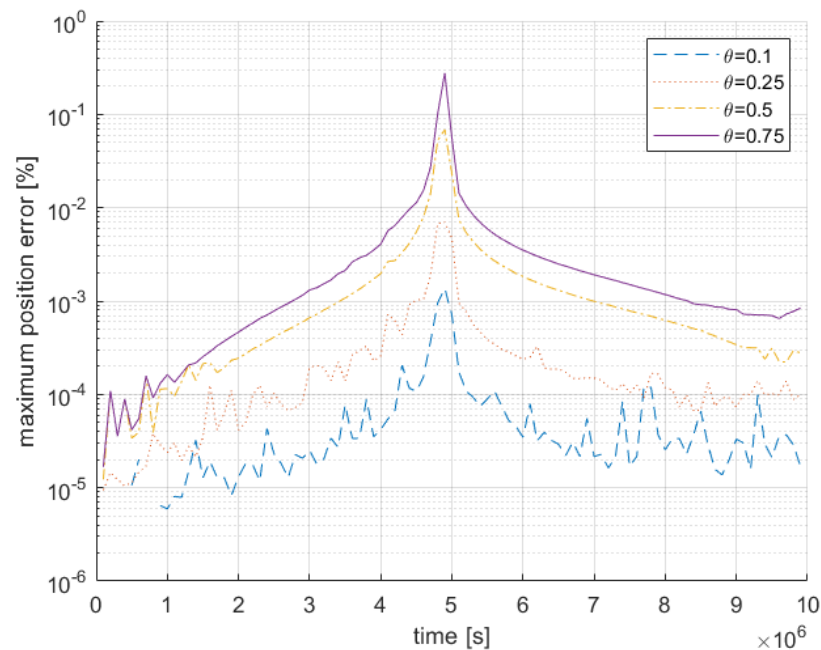# Implementation and methods

Gravitational dynamics: performance

Computational time

Accuracy (depends on $\theta_{accuracy}$)

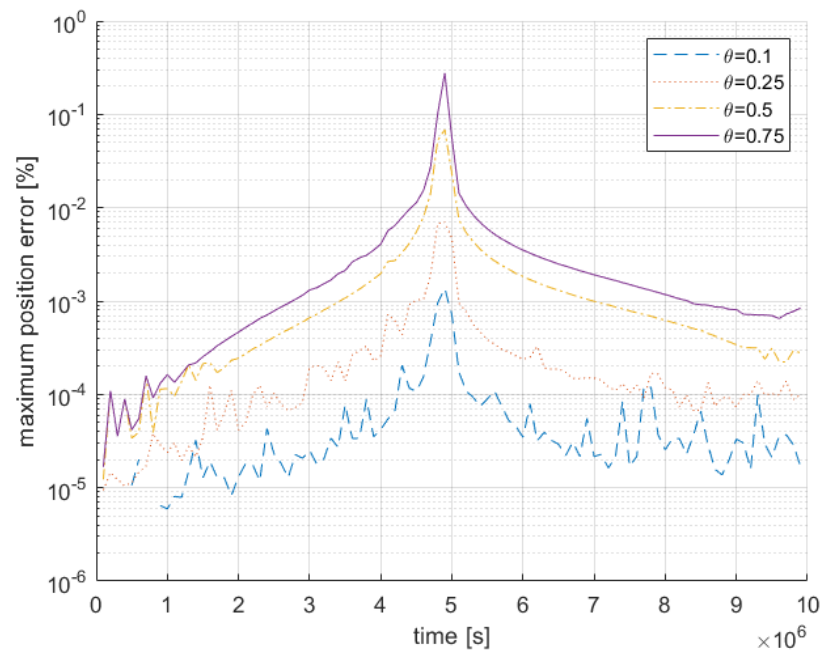# Implementation and methods

Gravitational dynamics: performance

Computational time

N<1000 : direct N2
N>1000 : BH-GPU

N^2

BH-GPU

θ

$N \log(N)$

time to compute time step [ms]

N bodies

Accuracy (depends on $\boldsymbol{\theta_{accuracy}}$)

maximum position error [%]

time [s]

$\times 10^6$

- – – $\theta=0.1$
- ⋯⋯ $\theta=0.25$
- –·–· $\theta=0.5$
- —— $\theta=0.75$

Implementation and methods

Software architecture

N-body gravitational problem with contact and collisions

Gravitational dynamics

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- central field (shape-based model)
  - Polyhedron (mesh)

Contact dynamics

- 6 DOF rigid body dynamics
- bodies of arbitrary shape
- collision detection
- contact methods
  - hard-body, constraint-based
  - soft-body, penalty-based
  - constraint-based with compliance and damping

Implementation and methods

Rigid-body dynamics

N bodies, each with

- position $\boldsymbol{r}_i$
- rotation quaternion $\boldsymbol{\rho}_i$
- velocity $\dot{\boldsymbol{r}}_i$
- angular velocity $\boldsymbol{\omega}_i$

Generalized coordinates
$$\boldsymbol{q} = \left\{\boldsymbol{r}_i^T, \boldsymbol{\rho}_i^T\right\}^T \in \mathbb{R}^{7N}$$
$$\boldsymbol{v} = \left\{\dot{\boldsymbol{r}}^T{}_i, \boldsymbol{\omega}_i^T\right\}^T \in \mathbb{R}^{6N}$$

- mass m_i
- tensor of inertia $\mathbf{I}_i$

$$\mathbf{M} = [m_i] \in \mathbb{R}^{6N \times 6N}$$
$$\mathbf{J} = [\mathbf{I}_i] \in \mathbb{R}^{6N \times 6N}$$

- collision surface Ω_i

Shape:
- Triangulated mesh
- Convex hull
- Common geometry (sphere, box, cone,…)
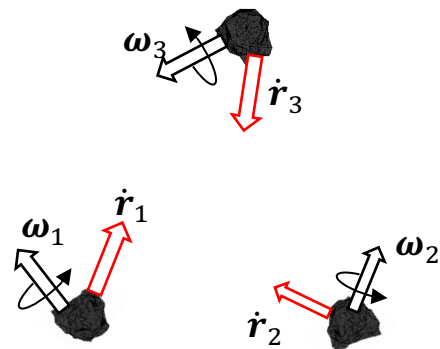
Implementation and methods

Contact dynamics: non-smooth dynamics (NSC)

- Equations of motion are formulated as Differential Variational Inequalities (DVI)
- Hard-body model
- Complementarity-based
- Impulse-momentum formulation
- Suitable for problems with discontinuities (rigid contacts)

$$\begin{cases} \gamma \text{ (contact) as solution of CCP} \\ \boldsymbol{v_{n+1}} = f(\boldsymbol{q}, \boldsymbol{v}, t, \gamma) \\ \boldsymbol{q_{n+1}} = g(\boldsymbol{q}, \boldsymbol{v}) \end{cases}$$

Parameters of the model:
- Friction (static, dynamic, spinning)
- Cohesion (value and constitutive model)
- Restitution coefficient

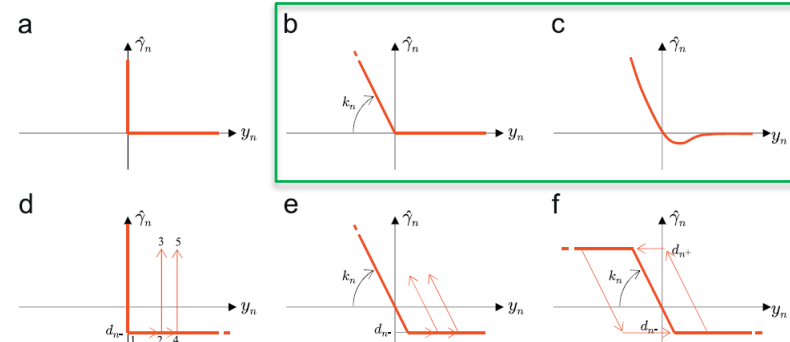Credits: Tasora et al 2013
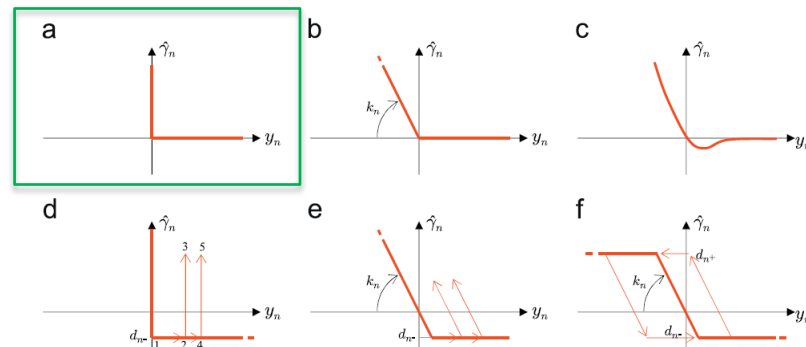
Fig. 1. Basic constitutive relations for normal reaction.

Implementation and methods

Contact dynamics: smooth dynamics (SMC)

- Equations of motion are formulated as Differential Algebraic equations (DAE)

$$\begin{cases} \dot{\boldsymbol{x}} = f(\boldsymbol{x}, t) \\ \boldsymbol{g}(\boldsymbol{x}, t) = 0 \end{cases}$$

ODE + AE (kinematic constraint)

- Soft-body model (DEM)
- Penalty-based
- Force-acceleration formulation
- Suitable for problems with no discontinuities (no rigid contacts)

Credits: Tasora et al 2013

Parameters of the model:

- Friction (static, dynamic, spinning)
- Cohesion (value and constitutive model)
- {Young modulus, Poisson ratio, restitution coefficient} or {stiffness and damping (normal and tangential)} and constitutive model (Hooke, Hertz)

In this case stiffness and damping are estimated based on constitutive law of material

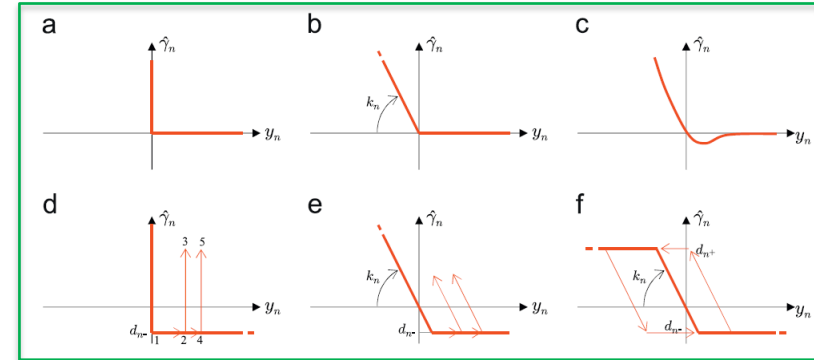**Fig. 1.** Basic constitutive relations for normal reaction.

Implementation and methods

Contact dynamics: hybrid model

- Equations of motion are formulated as Differential Variational Inequalities (DVI)
- Soft-body model (compliance and damping)
- Complementarity-based
- Impulse-momentum formulation
- Suitable for problems with discontinuities

$$\begin{cases} \gamma \text{ (contact) as solution of CCP} \\ \boldsymbol{v_{n+1}} = f(\boldsymbol{q}, \boldsymbol{v}, t, \gamma) \\ \boldsymbol{q_{n+1}} = g(\boldsymbol{q}, \boldsymbol{v}) \end{cases}$$

Parameters of the model:

- Friction (static, dynamic, spinning)
- Cohesion (value and constitutive model)
- Restitution coefficient
- Stiffness and damping (normal, tangential, rolling, spinning), rolling friction and constitutive model

Credits: Tasora et al 2013

Fig. 1. Basic constitutive relations for normal reaction.

Implementation and methods

Contact dynamics: summary

		NSC	SMC	Hybrid
Formulation	Equations of motion	DVI	DAE	DVI
	Contact model	hard	soft	soft
Performance	Computational time (single time step)	🟥	🟩	🟥
	Size of time step	🟩	🟥	🟨
	Reproducing non-rigid contact dynamics	🟥	🟩	🟩
	Handling complex shapes	🟩	🟥	🟩

Implementation and methods

Contact dynamics: tuning the parameters

Hybrid model
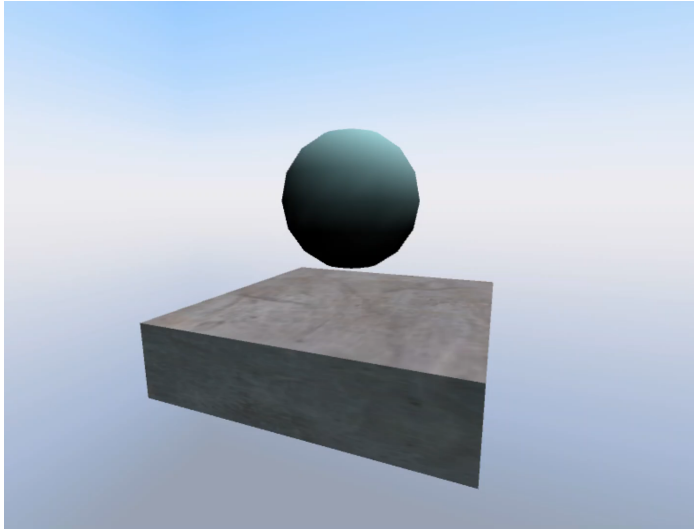
SMC (DEM)
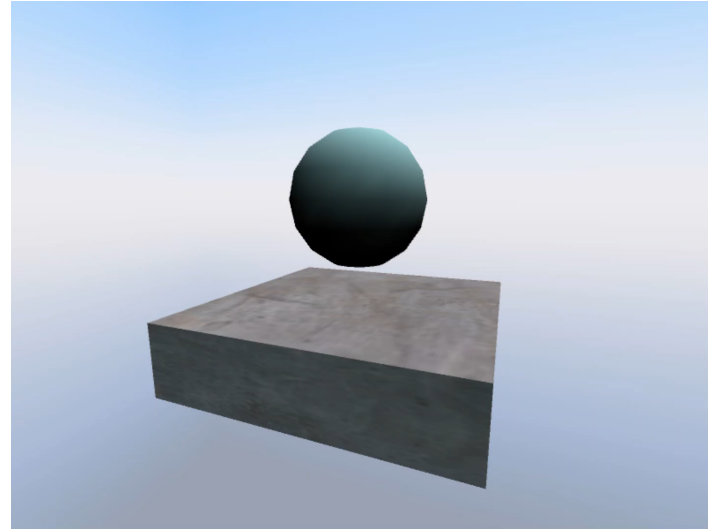
Small body environment

ASTEROID MODEL

Credits: JAXA/Hayabusa 2

SURFACE AND
GRANULAR TERRAIN

Credits: JAXA/Hayabusa 2/Minerva-II1-B

OTHER EFFECTS

SRP
dust/particles
ejecta
coma

Small body environment

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SRP
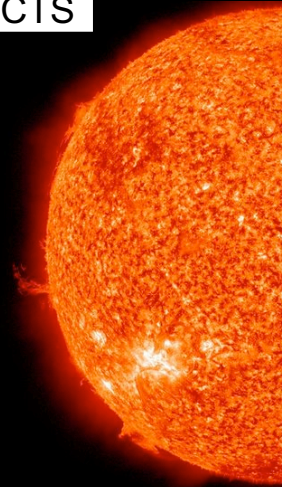
dust/particles

ejecta

coma

Asteroid model

Shape and internal structure

Shape

- equilibrium shape
- given mesh

Internal structure

- full rubble-pile
- monolithic core

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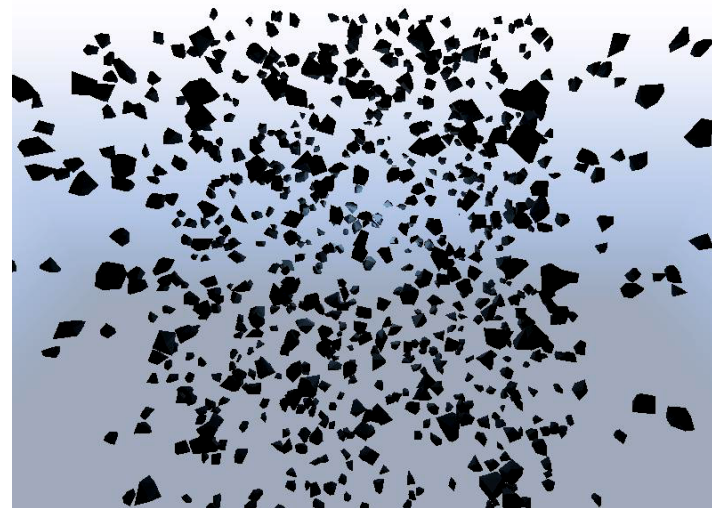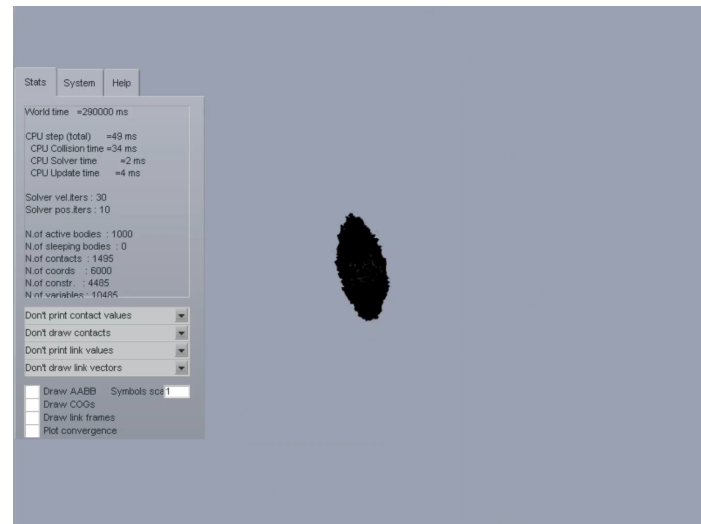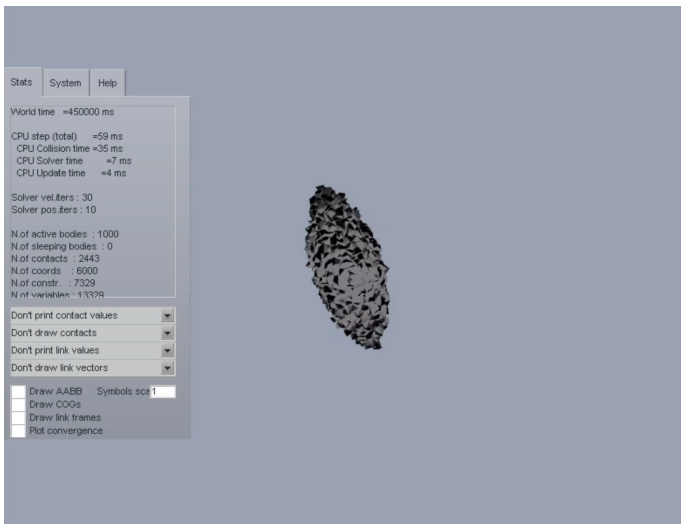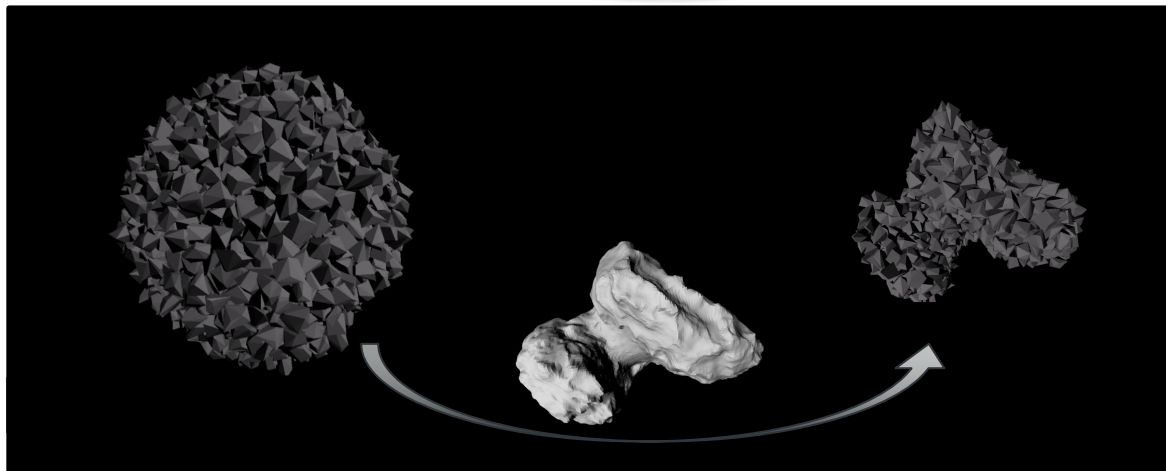
Asteroid model

Monolithic core

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Small body environment

ASTEROID MODEL

Credits: JAXA/Hayabusa 2

SURFACE AND
GRANULAR TERRAIN

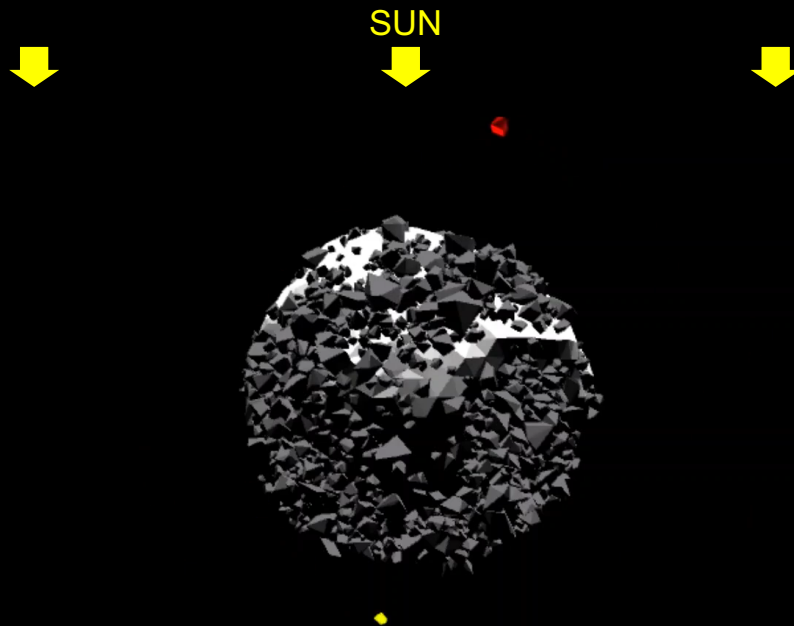Credits: JAXA/Hayabusa 2/Minerva-II1-B

OTHER EFFECTS

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Other effects: solar radiation pressure
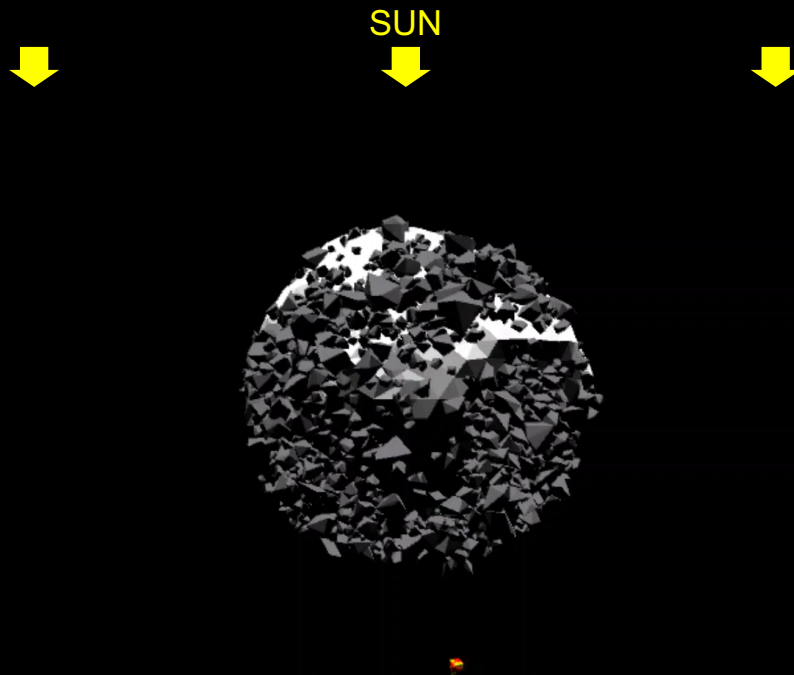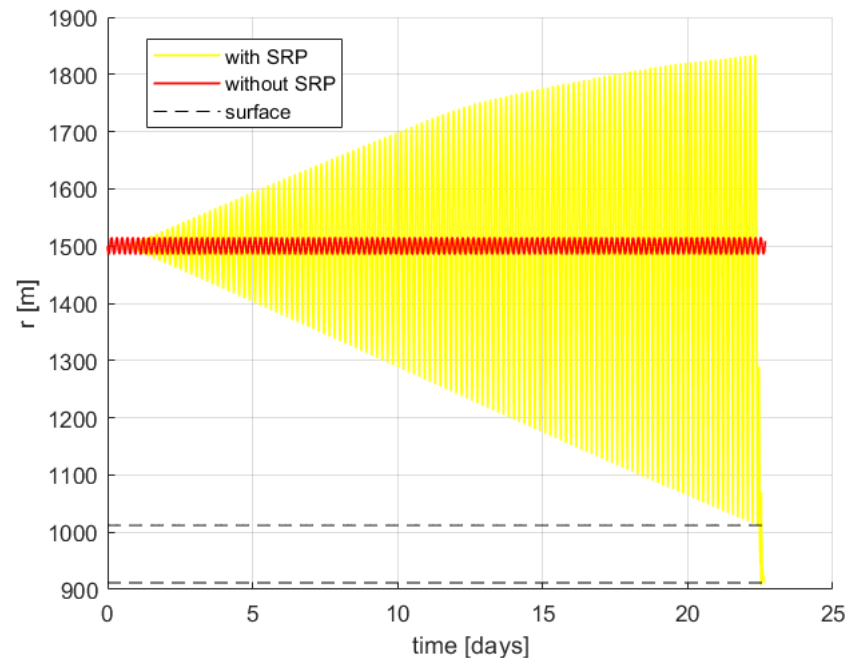

SUN

with SRP
with no SRP

j p l . n a s a . g o v

Other effects: solar radiation pressure
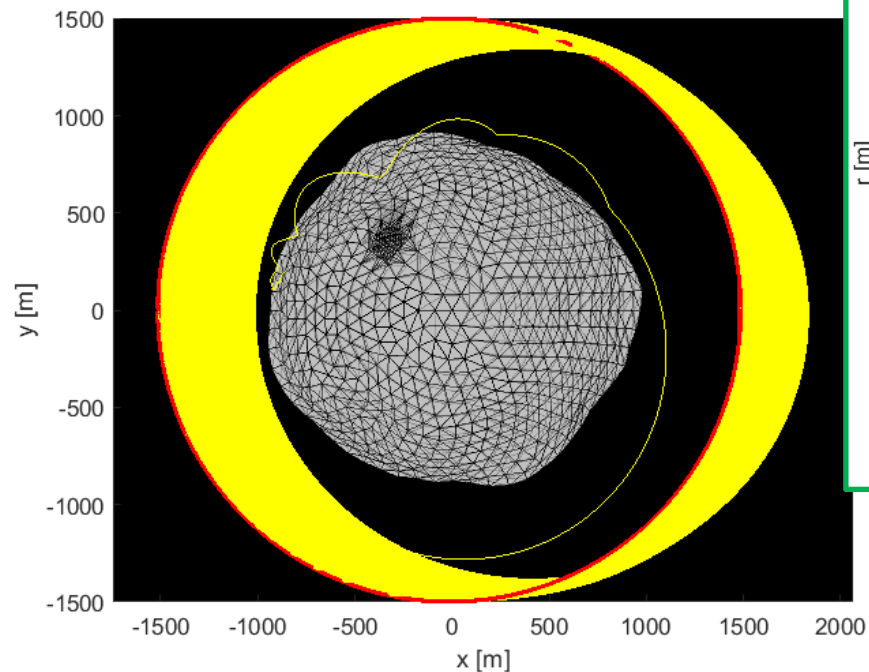

SUN

with SRP
with no SRP

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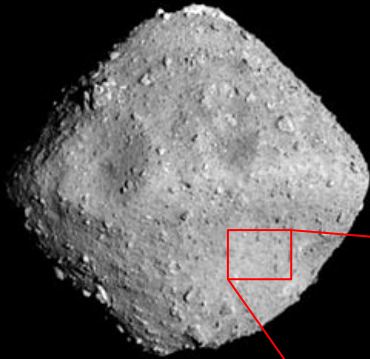# Other effects: solar radiation pressure

Validation scenario
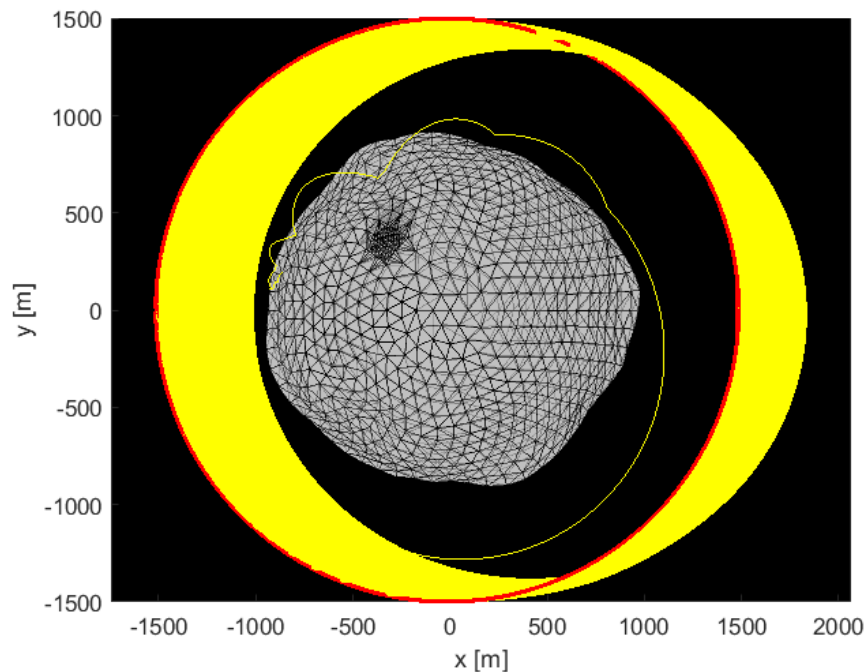
Other effects: solar radiation pressure
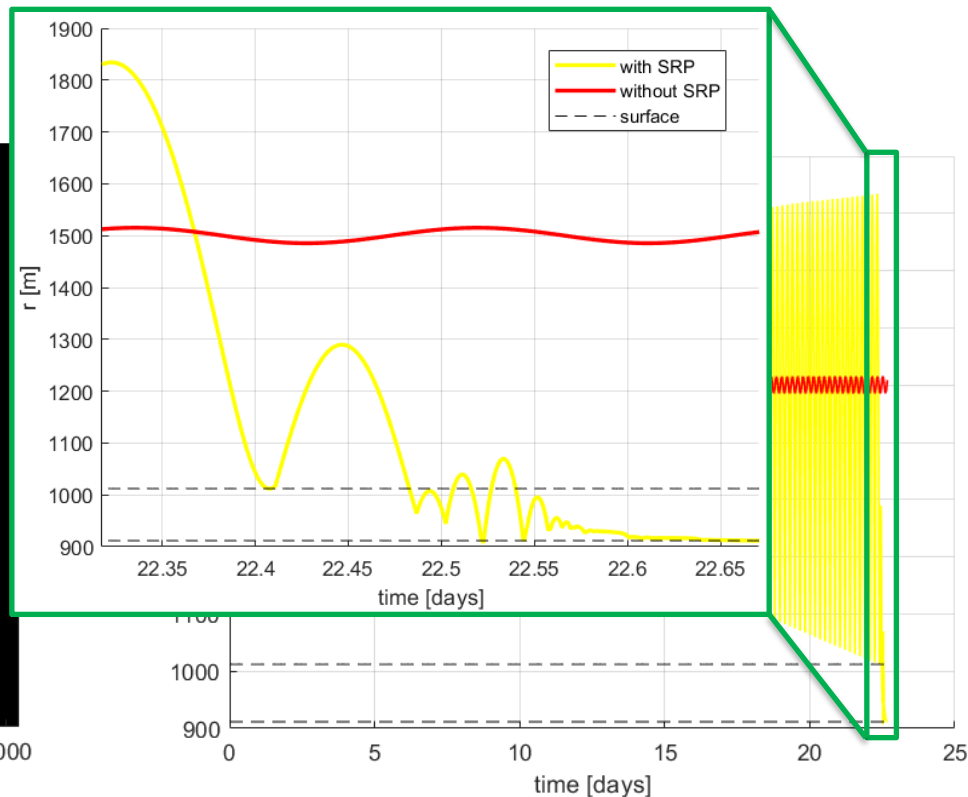
Validation scenario

Small body environment

ASTEROID MODEL

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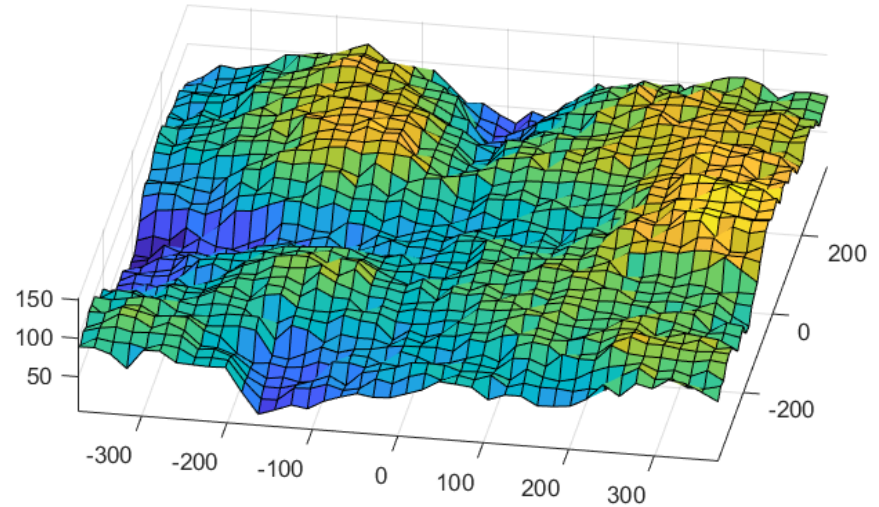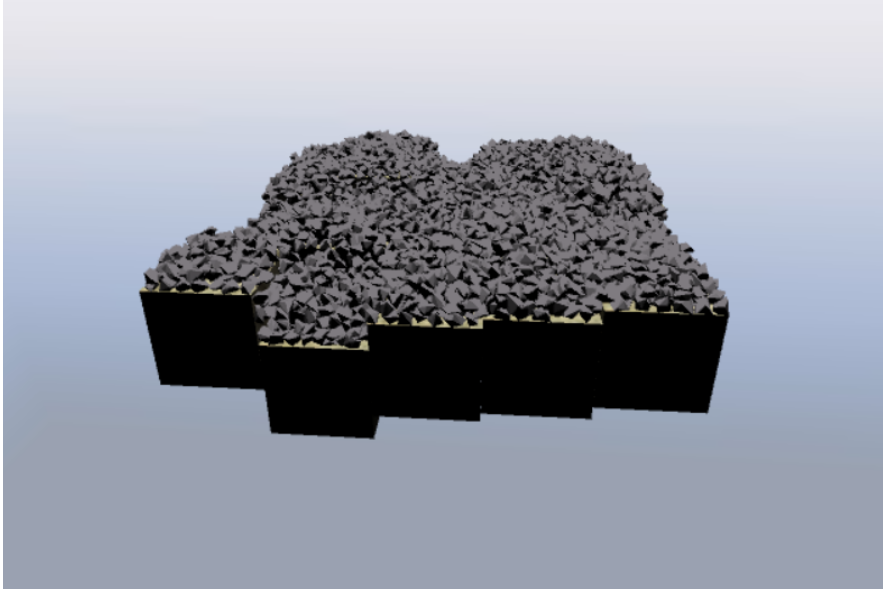SURFACE AND GRANULAR TERRAIN

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OTHER EFFECTS

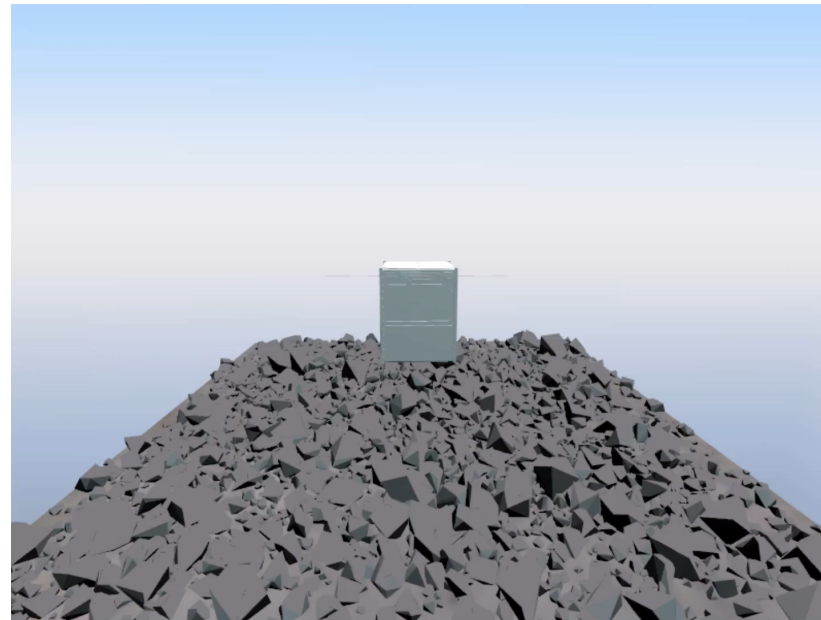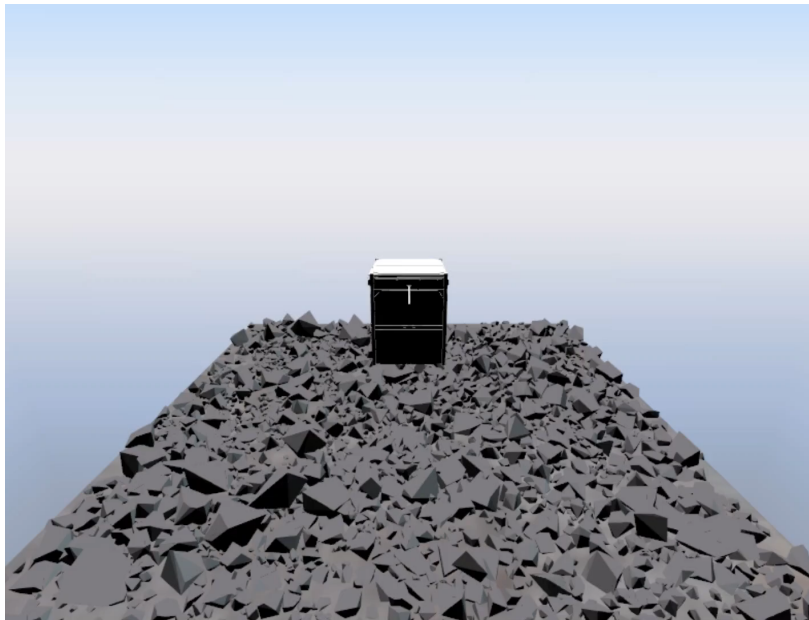SRP
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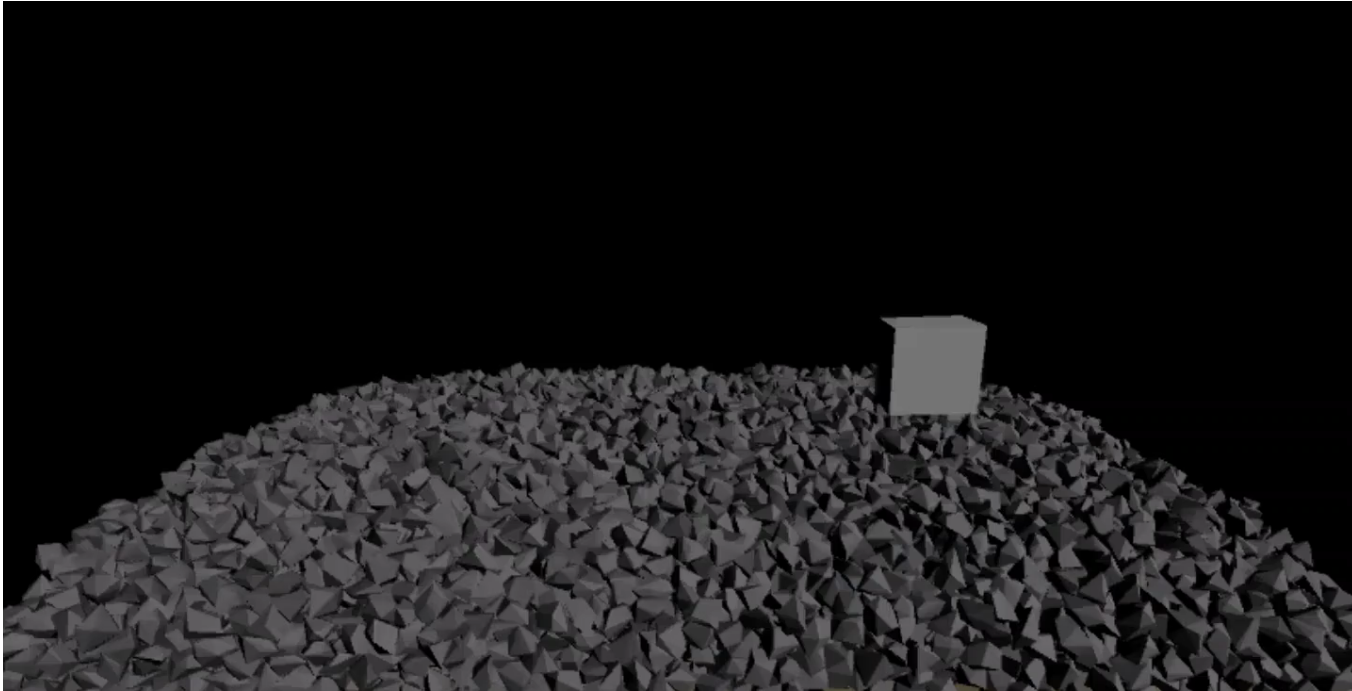# Surface and granular terrain

Creation of terrain

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Surface and granular terrain

Lander touch down

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Surface and granular terrain

Hopper

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Conclusion

FINAL HIGHLIGHTS

- Handles complex-shaped bodies
- State-of-the-art methods for gravitational dynamics: Barnes-Hut parallel GPU
- State-of-the-art methods for contact dynamics: both hard- and soft-contact models
- Great flexibility of models/methods and implementation

FUTURE WORK AND ONGOING COLLABORATIONS

- Go on with validation/benchmarking and developing effort (with Chrono::Engine team, Univ. Parma)
- Rubble pile aggregation / reconfiguration (with OCA)
- Lander/soil interaction and lander/rover mobility
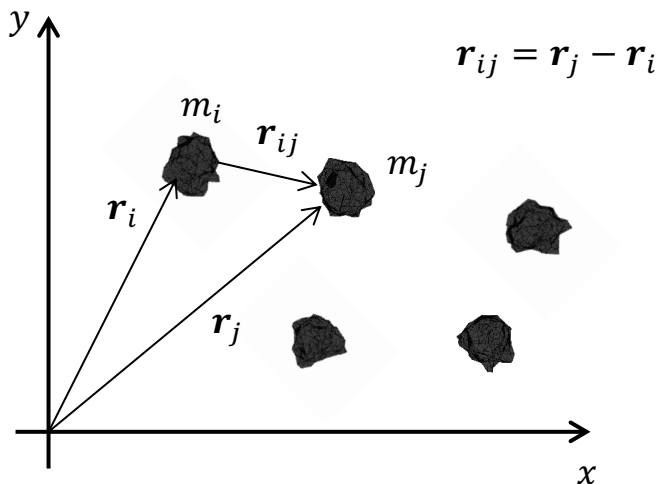- Planetary rings dynamics
- Rubble pile gravity field

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Backup

Implementation and methods

Gravitational dynamics: direct N-to-N integration

$$\boldsymbol{r}_{ij} = \boldsymbol{r}_j - \boldsymbol{r}_i$$

N equations of motion

$$m_i \, \ddot{\boldsymbol{r}}_i = G \sum_{j=1, j \neq i}^{N} \frac{m_i \, m_j}{r_{ij}^3} \boldsymbol{r}_{ij}$$

Features of the dynamical system
- No analytical solution for the gravitational motion of N bodies
- Highly non-linear (chaotic) behavior
- Strong dependency on initial conditions
- Slow dynamics: characteristic time
  $T \sim \frac{1}{\sqrt{G\rho}}$ (with $G = 6.67 \cdot 10^{-11} \frac{m^3}{kg \, s^2}$)

Features of the numerical problem
- Initial value problem
- Integration time step can be big
  $dt < \frac{T}{2} = \frac{1}{2\sqrt{G\rho}}$
  ($dt \sim 10^3 \, s$ for typical asteroids densities)

Implementation and methods

Numerical integration: available methods

Non-smooth dynamics (NSC)

Equations of motion are formulated as <u>Differential Variational Inequalities (DVI)</u>

Smooth dynamics (SMC)

Equations of motion are formulated as a <u>Differential Algebraic Equations (DAE)</u>

- Time-steppers:
  - Symplectic methods (semi-implicit Euler, leapfrog) → Suited for gravitational problem
  - Runge Kutta methods (RK45, explicit Euler, implicit Euler, trapezoidal, Heun) → Higher order
  - Newmark, Hilber-Hughes-Taylor → Suited for FEA problems

- Solvers:
  - Iterative solvers
  - Direct solvers

Most commonly used: good for both DVI and DAE problems